



Wydział Mechaniczny Energetyki i Lotnictwa
Zakład Wytrzymałości Materiałów i Konstrukcji



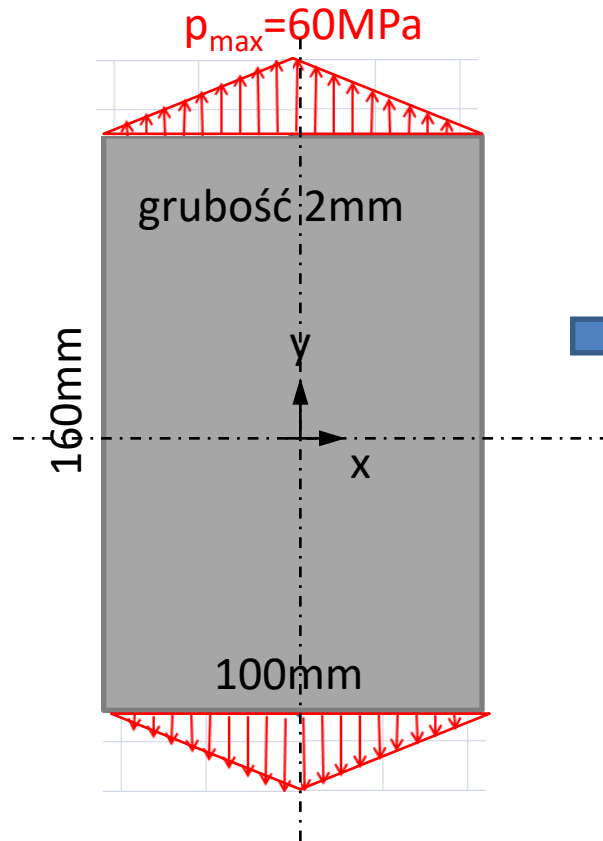
Metoda elementów skończonych (MES1)

Wykład 1d. Tarcza 2D modelowana elementami CST

(przypomnienie z MES1)

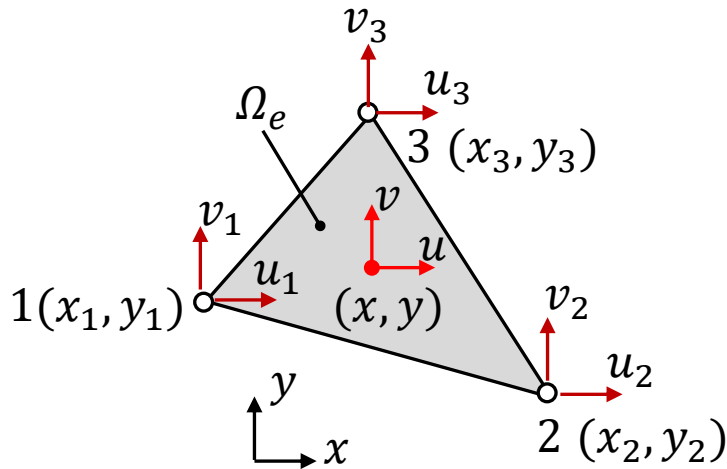
10.2024

Model prostokątnej tarczy



**Model ćwiartki tarczy
o dwóch elementach skończonych**

Przypomnienie dla elementu CST w stanie PSN



funkcje kształtu = znormalizowane pola:

$$N_1(x, y) = \frac{A_1(x, y)}{A_e} = \frac{1}{2A_e} (a_1 + b_1x + c_1y)$$

$$N_2(x, y) = \frac{A_2(x, y)}{A_e} = \frac{1}{2A_e} (a_2 + b_2x + c_2y)$$

$$N_3(x, y) = \frac{A_3(x, y)}{A_e} = \frac{1}{2A_e} (a_3 + b_3x + c_3y)$$

gdzie:

$$\begin{aligned} a_1 &= x_2y_3 - x_3y_2 & ; & & a_2 &= x_3y_1 - x_1y_3 & ; & & a_3 &= x_1y_2 - x_2y_1 \\ b_1 &= y_2 - y_3 & ; & & b_2 &= y_3 - y_1 & ; & & b_3 &= y_1 - y_2 \\ c_1 &= x_3 - x_2 & ; & & c_2 &= x_1 - x_3 & ; & & c_3 &= x_2 - x_1 \end{aligned}$$

$$\begin{aligned} a_i &= x_jy_k - x_ky_j \\ b_i &= y_j - y_k \\ c_i &= x_k - x_j \end{aligned}$$

macierz odkształcenie-przemieszczenie

$$[B] = \frac{1}{2A_e} \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix}$$

$$[D] = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix}$$

macierz
konstrytuwna dla PSN:

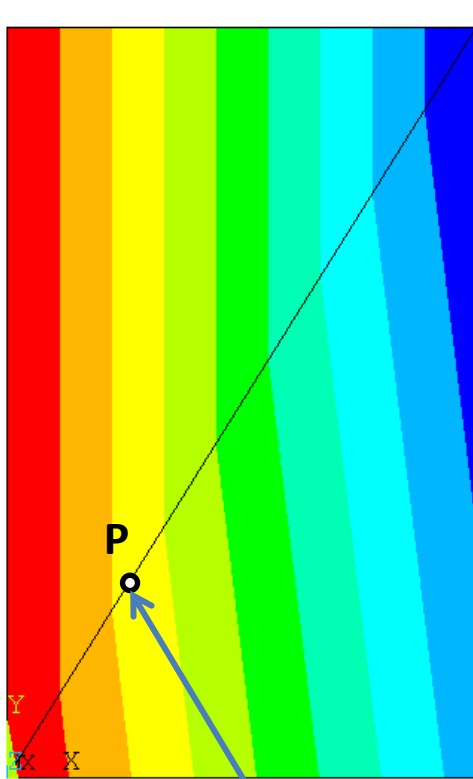
lokalna macierz sztywności elementu:

$$[k]_e = A_e t_e [B]^T [D] [B]$$

6×6 6×3 3×3 3×6

Przemieszczenia w punkcie P na granicy elementów

UX displacement

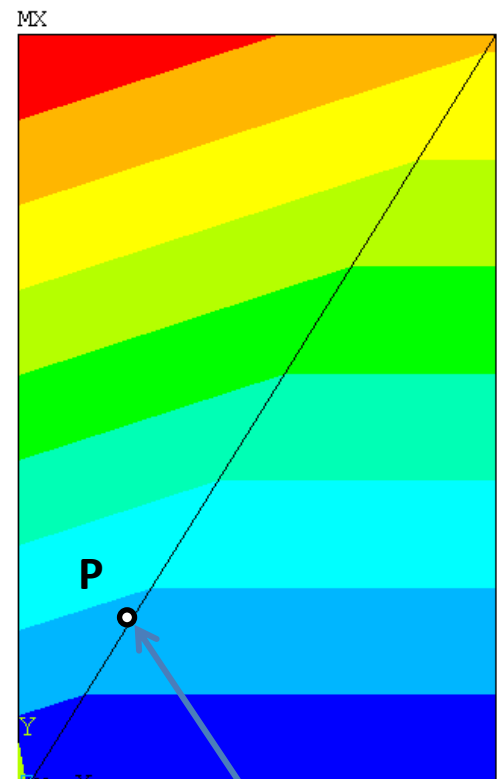


PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMN =-.007784
-.007784
-.006919
-.006054
-.005189
-.004324
-.00346
-.002595
-.00173
-.865E-03
0

-0.00195

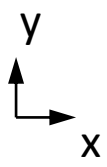
[mm]

UY displacement



PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMX =.038165
0
.004241
.008481
.012722
.016962
.021203
.025444
.029684
.033925
.038165

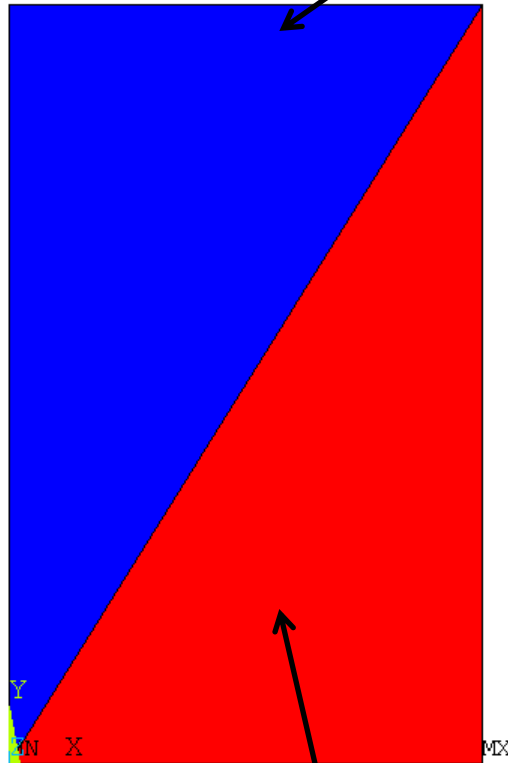
0.0076



Odkształcenia w punkcie P na granicy elementów

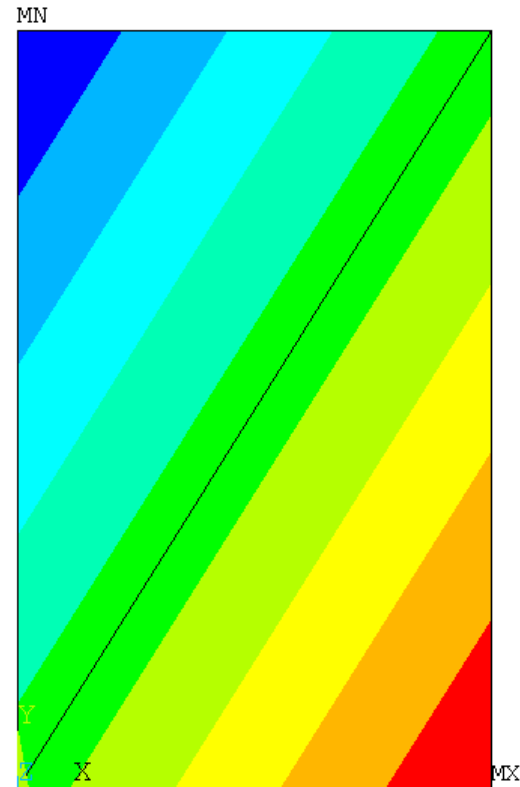


$\epsilon_2 =$	-0.000155682
	0.000477066
	-0.000155183



ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.038165
 SMN =-.156E-03
 SMX =-.130E-03

■	-.156E-03
■	-.153E-03
■	-.150E-03
■	-.147E-03
■	-.144E-03
■	-.141E-03
■	-.139E-03
■	-.136E-03
■	-.133E-03
■	-.130E-03

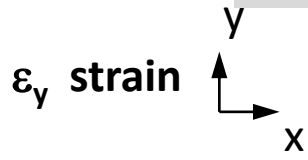


PLOT NO. 12
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.038165
 SMN =-.156E-03
 SMX =-.130E-03

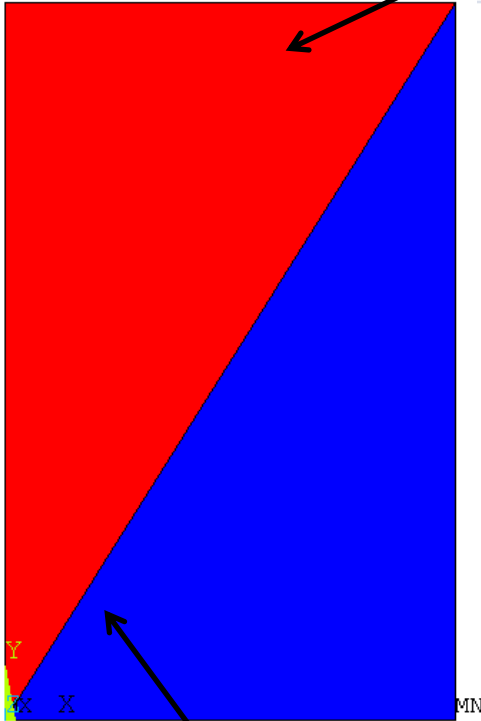
■	-.156E-03
■	-.153E-03
■	-.150E-03
■	-.147E-03
■	-.144E-03
■	-.141E-03
■	-.139E-03
■	-.136E-03
■	-.133E-03
■	-.130E-03

$\epsilon_1 =$	-0.000130032
	0.000380077
	-1.60313E-05

Odkształcenia w punkcie P na granicy elementów



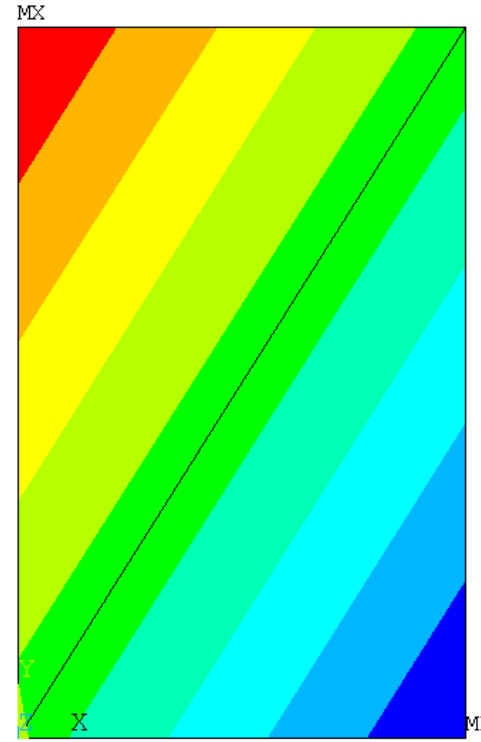
$\epsilon_2 =$	-0.000155682
	0.000477066
	-0.000155183



ELEMENT SOLUTION

STEP=1
SUB =1
TIME=1
EPELY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =.380E-03
SMX =.477E-03

Blue	.380E-03
Light Blue	.391E-03
Cyan	.402E-03
Light Green	.412E-03
Green	.423E-03
Light Yellow	.434E-03
Yellow	.445E-03
Orange	.456E-03
Red	.466E-03
Dark Red	.477E-03



PLOT NO. 13

NODAL SOLUTION

STEP=1
SUB =1
TIME=1
EPELY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMN =.380E-03
SMX =.477E-03

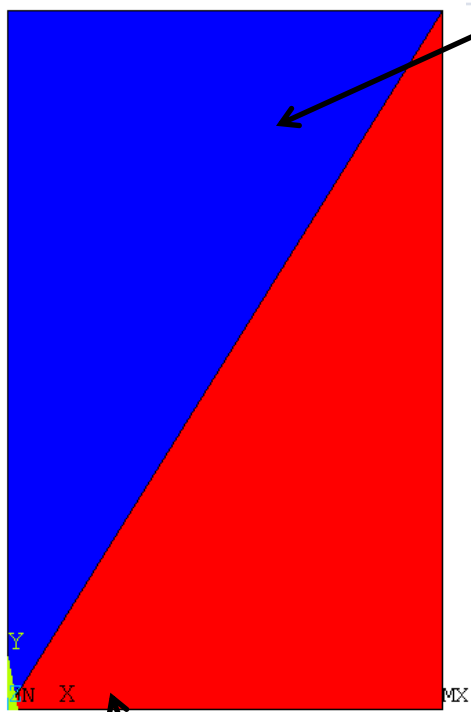
Blue	.380E-03
Light Blue	.391E-03
Cyan	.402E-03
Light Green	.412E-03
Green	.423E-03
Light Yellow	.434E-03
Yellow	.445E-03
Orange	.456E-03
Red	.466E-03
Dark Red	.477E-03

$\epsilon_1 =$	-0.000130032
	0.000380077
	-1.60313E-05

Odształcenia w punkcie P na granicy elementów



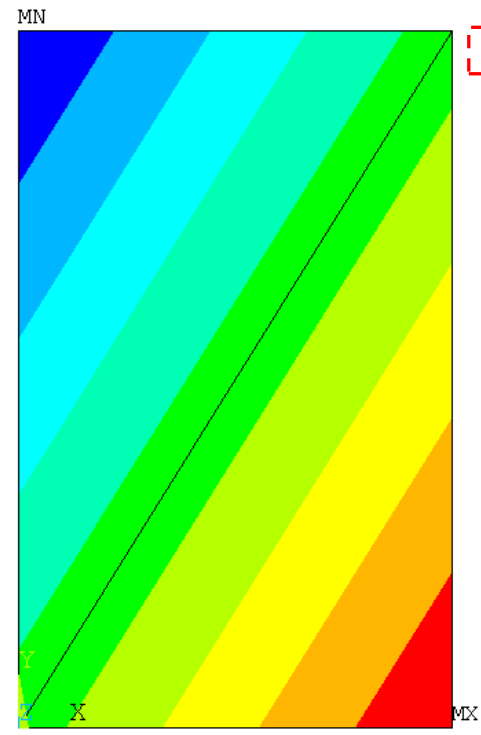
$\varepsilon_2 =$	-0.000155682
	0.000477066
	-0.000155183



ELEMENT SOLUTION

STEP=1
SUB =1
TIME=1
EPELXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-.155E-03
SMX =-.160E-04

Blue	-.155E-03
Light Blue	-.140E-03
Cyan	-.124E-03
Light Green	-.109E-03
Green	-.933E-04
Yellow-Green	-.779E-04
Yellow	-.624E-04
Orange	-.470E-04
Red	-.315E-04
Dark Red	-.160E-04



PILOT NO. 14

NODAL SOLUTION

STEP=1
SUB =1
TIME=1
EPELXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMN =-.155E-03
SMX =-.160E-04

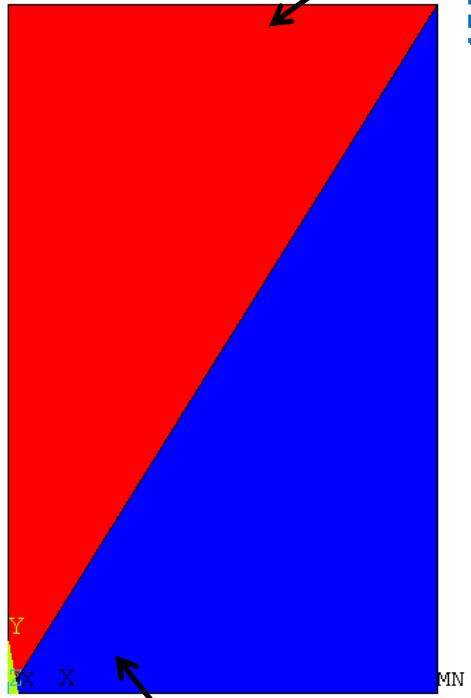
Blue	-.155E-03
Light Blue	-.140E-03
Cyan	-.124E-03
Light Green	-.109E-03
Green	-.933E-04
Yellow-Green	-.779E-04
Yellow	-.624E-04
Orange	-.470E-04
Red	-.315E-04
Dark Red	-.160E-04

$\varepsilon_1 =$	-0.000130032
	0.000380077
	-1.60313E-05

Naprężenia w punkcie P na granicy elementów



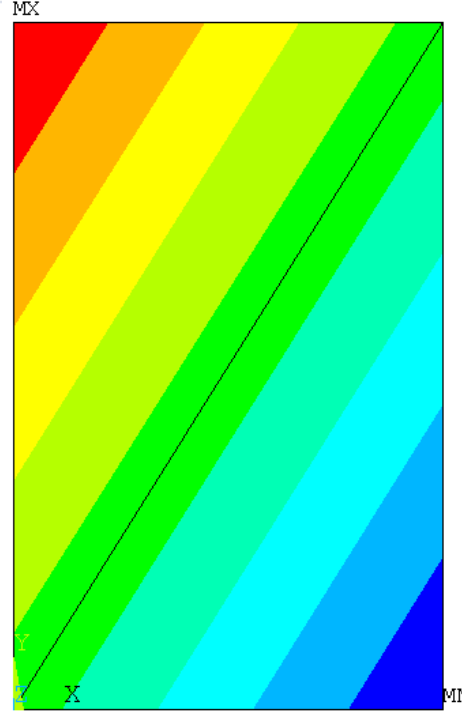
$\sigma_2 =$	0.263	MPa
	33.48	MPa
	-4.074	MPa



ELEMENT SOLUTION

STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-.263011
SMX =.263011

Blue	-.263011
Light Blue	-.204564
Cyan	-.146117
Light Green	-.08767
Green	-.029223
Yellow-Green	.029223
Yellow	.08767
Orange	.146117
Light Orange	.204564
Red	.263011



PLOT NO. 9

NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMN =-.263011
SMX =.263011

Blue	-.263011
Light Blue	-.204564
Cyan	-.146117
Light Green	-.08767
Green	-.029223
Yellow-Green	.029223
Yellow	.08767
Orange	.146117
Light Orange	.204564
Red	.263011

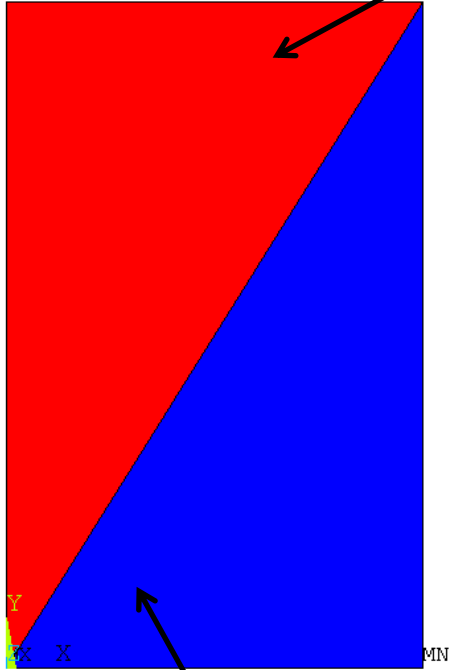
$\sigma_1 =$	-0.263	MPa
	26.52	MPa
	-0.421	MPa

[MPa]

Naprężenia w punkcie P na granicy elementów

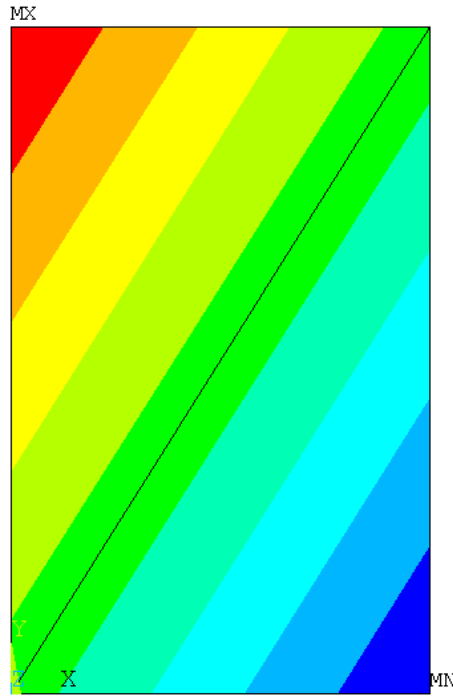


$\sigma_2 =$	0.263	MPa
	33.48	MPa
	-4.074	MPa



ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =26.518
SMX =33.482

26.518
27.292
28.065
28.839
29.613
30.387
31.161
31.935
32.708
33.482



PLOT NO = 10
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMN =26.518
SMX =33.482

26.518
27.292
28.065
28.839
29.613
30.387
31.161
31.935
32.708
33.482

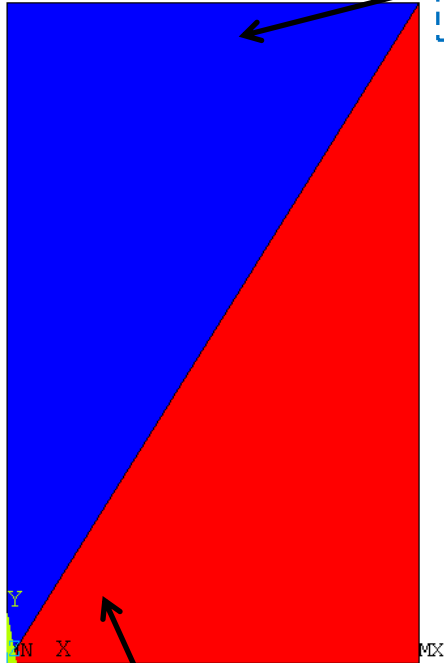
$\sigma_1 =$	-0.263	MPa
	26.52	MPa
	-0.421	MPa

[MPa]

Naprężenia w punkcie P na granicy elementów

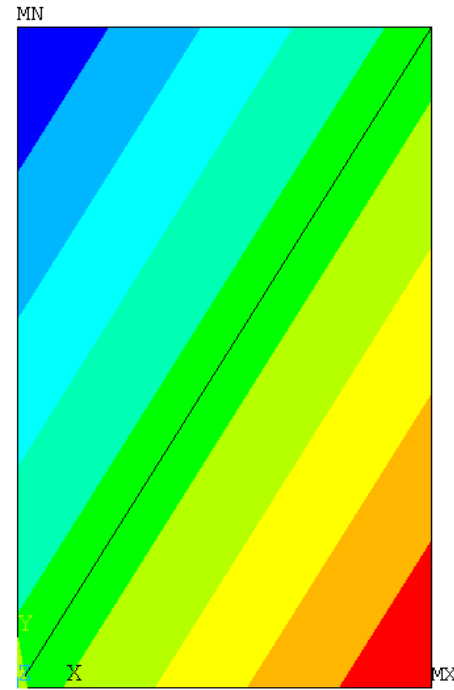


	0.263	MPa
$\sigma_2 =$	33.48	MPa
	-4.074	MPa



ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-4.074
SMX =-.420818

Blue	-4.074
Light Blue	-3.668
Cyan	-3.262
Green	-2.856
Light Green	-2.45
Yellow-Green	-2.044
Yellow	-1.638
Orange	-1.233
Red-Orange	-.826679
Red	-.420818



PLOT NO. 11
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMN =-4.074
SMX =-.420818

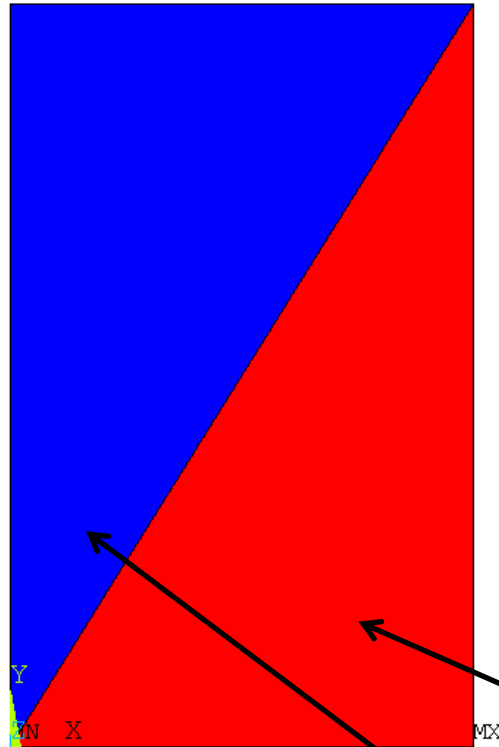
Blue	-4.074
Light Blue	-3.668
Cyan	-3.262
Green	-2.856
Light Green	-2.45
Yellow-Green	-2.044
Yellow	-1.638
Orange	-1.233
Red-Orange	-.826679
Red	-.420818

	-0.263	MPa
$\sigma_1 =$	26.52	MPa
	-0.421	MPa

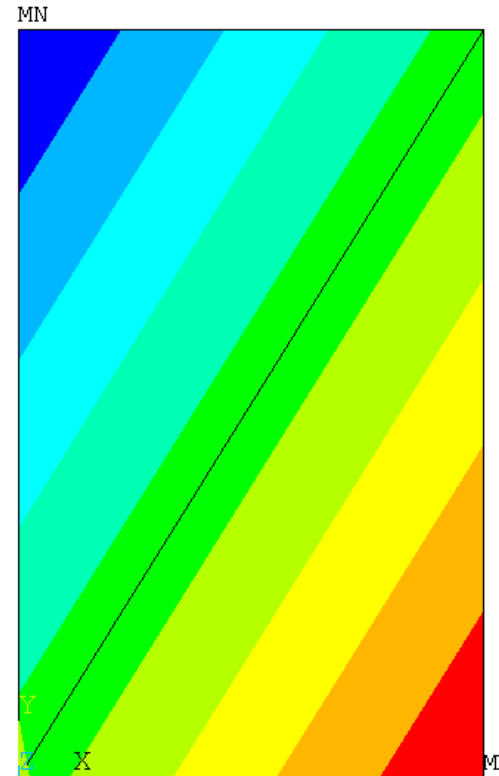
[MPa]

Odształcenia w punkcie P na granicy elementów

ε_z strain



PLOT NO. 6
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELZ (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.038165
 SMN =-.161E-03
 SMX =-.125E-03
 -.161E-03
 -.157E-03
 -.153E-03
 -.149E-03
 -.145E-03
 -.141E-03
 -.137E-03
 -.133E-03
 -.129E-03
 -.125E-03



PLOT NO. 15
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELZ (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.038165
 SMN =-.161E-03
 SMX =-.125E-03
 -.161E-03
 -.157E-03
 -.153E-03
 -.149E-03
 -.145E-03
 -.141E-03
 -.137E-03
 -.133E-03
 -.129E-03
 -.125E-03

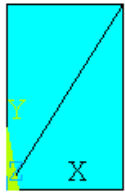
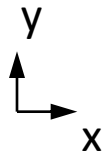
$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = \begin{cases} -\frac{1}{3.7 \cdot 10^4}(-0.263 + 26.52) = -0.125 \cdot 10^{-3} \\ -\frac{1}{3.7 \cdot 10^4}(0.263 + 33.48) = -0.161 \cdot 10^{-3} \end{cases}$$

Wpływ dyskretyzacji na jakość wyników

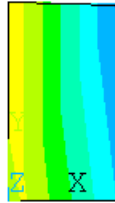
DOF Solution

UX

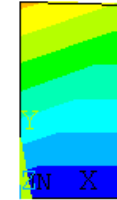
UY



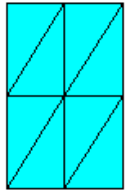
2



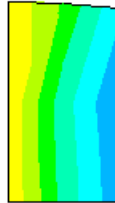
PLOT NO. 1
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.047134
 SMN =-.008912
 SMX =.00282



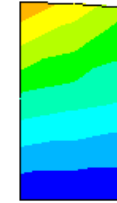
PLOT NO. 2
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.047134
 SMX =.047134



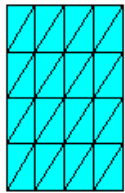
8



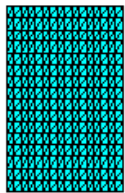
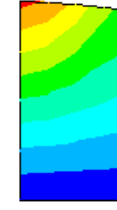
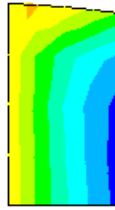
-.008912
 -.007609
 -.006305
 -.005001
 -.003698
 -.002394
 -.00109
 .213E-03
 .001517
 .00282



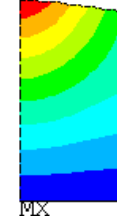
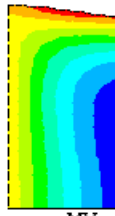
0
 .005237
 .010474
 .015711
 .020949
 .026186
 .031423
 .03666
 .041897
 .047134



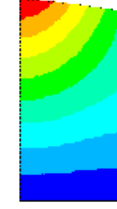
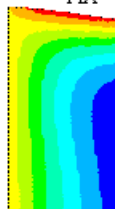
32



512



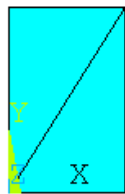
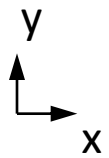
8000



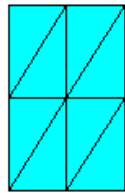
[mm]

Wpływ dyskretyzacji na jakość wyników

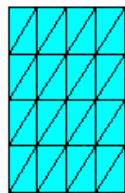
Horizontal stress σ_x



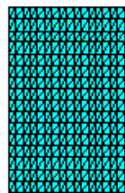
2



8



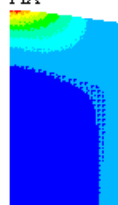
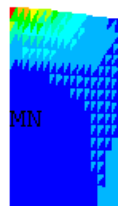
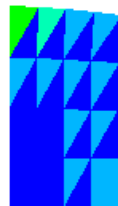
32



512



8000



PLOT NO. 3
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.047134
 SMN =-4.88
 SMX =35.587

Blue	-4.88
Light Blue	-.383303
Cyan	4.113
Light Green	8.609
Green	13.106
Yellow-Green	17.602
Yellow	22.098
Orange	26.595
Red-Orange	31.091
Red	35.587



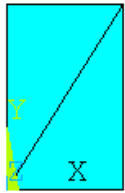
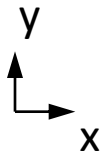
PLOT NO. 7
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.047134
 SMN =-4.784
 SMX =35.587

Blue	-4.784
Light Blue	-.298737
Cyan	4.187
Light Green	8.673
Green	13.159
Yellow-Green	17.644
Yellow	22.13
Orange	26.616
Red-Orange	31.102
Red	35.587

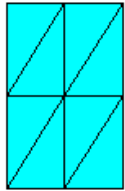
[MPa]

Wpływ dyskretyzacji na jakość wyników

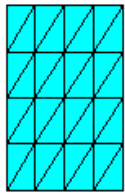
Vertical stress σ_y



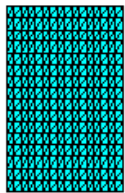
2



8



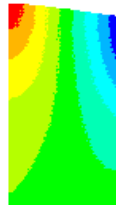
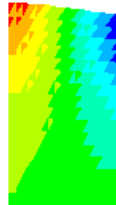
32



512



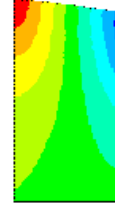
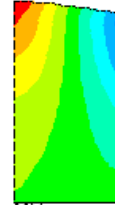
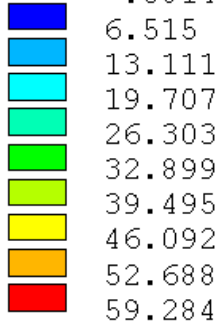
8000



PLOT NO. 4
ELEMENT SOLUTION

STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0

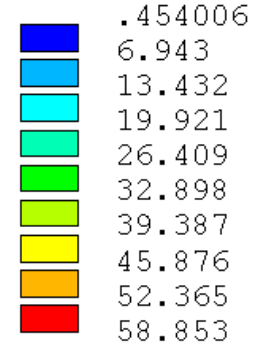
PowerGraphics
EFACET=1
DMX =.047134
SMN =-.081405
SMX =59.284



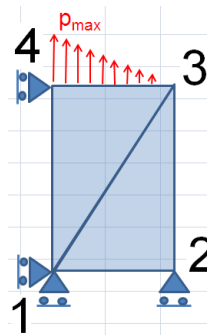
PLOT NO. 8
NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0

PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047134
SMN =.454006
SMX =58.853

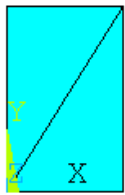
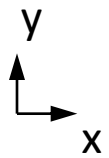


[MPa]

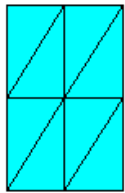


Wpływ dyskretyzacji na jakość wyników

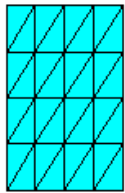
Shear stress τ_{xy}



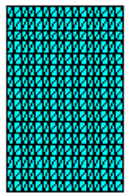
2



8



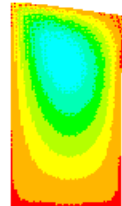
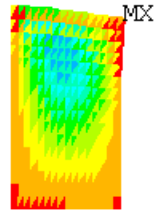
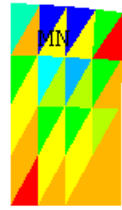
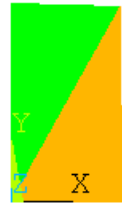
32



512

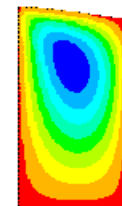
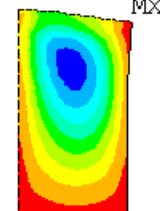
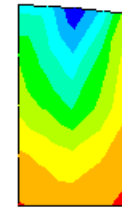
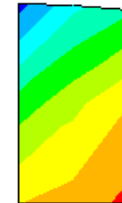


8000



PLOT NO. 5
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.047134
 SMN =-9.769
 SMX =1.102

Blue	-9.769
Light Blue	-8.561
Cyan	-7.353
Green	-6.145
Light Green	-4.938
Yellow-Green	-3.73
Yellow	-2.522
Orange	-1.314
Red-Orange	-.106324
Red	1.102



PLOT NO. 9
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.047134
 SMN =-7.077
 SMX =.408315

Blue	-7.077
Light Blue	-6.246
Cyan	-5.414
Green	-4.582
Light Green	-3.75
Yellow-Green	-2.919
Yellow	-2.087
Orange	-1.255
Red-Orange	-.423425
Red	.408315

[MPa]

Wpływ dyskretyzacji na jakość wyników

